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The Density of Stars of Different Spectral Types.

By

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With 1 figure.

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According to a previous article the Period of binary systems is given by the relation lag $P = -0.542 \log \bar{\varrho}_{\min} - 0.301$. Consequently the distribution of the periods of the binaries simultaneously must represent the distribution of the densities of the teneous components. For those eclipsing variables for which the spectral type of the teneous components is known, the distribution of these densities is investigated and compared with that of the Cepheids. With the eclipsing stars both the main sequence and the giant series are clearly indicated. The region occupied by the eclipsing stars is avoided by the Cepheids and conversely. The interval which separates the long and the short period Cepheids coincides with the region where the Cepheid belt intersects the giant series.

§ 1. Introduction.

In a recent article [1] evidence was given that the period of a spectroscopic binary is closely correlated to the mean density of its teneous component. Analytically this relation could be expressed by the equation:

$$\log P = -0.542 \log \bar{\varrho} - 0.301 \pm 0.147 \text{ (p. e.)} \qquad (1)$$

where P is the period expressed in days and $\bar{\varrho}$ the density of the teneous component ($\odot = 1$).

We may expect this same relation to hold for the other close binary systems such as the variables of the β Lyrae, Algol and W Ursa Majoris class. As a matter of fact several of such variables of which the spectroscopic elements are known, were included in the group of stars, from which the relation (1) was derived.

If the period of a narrow pair is determined by the density of its teneous component, we conclude that conversely the density of the teneous component is closely related to the period and can be computed from (1).

Therefore a study of the periods of the eclipsing variables of the different spectral types should reveal the distribution of the densities within each spectral type.

The distribution of the periods of eclipsing variables of the different spectral types has frequently been studied, but usually as the spectral type of the system the spectrum of the brighter component has been adopted. From our results however it would appear that the double systems are mainly characterised by the mean density respectively the spectral type of the teneous component. Consequently any statistical study of periods and densities should be based on the elements of this teneous component. Unfortunately with the majority of the eclipsing variables only one spectrum e. g. that of the brigther component has been observed and as a rule we do not know whether this is the dense or the teneous component. Even in those cases where both spectra have been observed, we usually do not know which spectrum is that of the dense and which that of the teneous component. Consequently in a statistical discussion of the periods of the eclipsing variables observed until now only a limited number can be used viz. only those pairs for which we can find the spectral type of the teneous component.

§ 2. The available Material.

From the new catalogue of variable stars [2] by L. B. Kukarkin and P. P. Parenago I have collected all β Lyrae and W Ursa Majoris variables for which one or both spectral types are given. With these variables we are reasonably certain that the two components closely resemble one another and that they have almost equal masses, densities and spectral types. These systems are enumerated in table I. For the sake of brevity only those data are given which are used in the following:

Table 1. *β-Lyrae and* W *Ursa Majoris System.*

Design	log P	Spect.	Design	log P	Spect.	Design	lcg P	Spect.
AA And	— 0.029	A5	GM Car	+ .185	B9	RZ Com	— .471	F5
AB And	— .479	G5	GW Car	+ .090	B5	SS Com	— .384	F5
AN And	+ .508	A7	RX Cas	+ 1.509	gG3 +	UU CrA	+ .350	A0
S Ant	— .188	A8			(gA5)	V Crt	— .154	A6
ST Aqr	— .108	F0	TX Cas	+ .467	B4	W Cru	+ 2.297	G0
SU Aqr	+ .017	A5	DO Cas	— .164	A2	ZZ Cru	+ .269	B8
OO Aql	— .295	G5	RR Cen	— .217	F2	WZ Cru	— .236	F0
V337 Aql	+ .436	B3	RZ Cen	+ .272	B2	CG Cru	— .200	F2
σ Aql	+ .290	B3	SV Cen	+ .220	B8	G0 Cru	— .144	A0
LR Ara	+ .182	B2	SZ Cen	+ .614	A2	V366 Cru	+ .009	A5
SX Aur	+ .083	B4	VZ Cen	+ .693	B2	V367 Cru	+ 1.270	F2
TT Aur	+ .154	B3	LW Cen	.000	B8	V388 Cru	— .066	A3
ZZ Aur	— .221	A7	LZ Cen	+ .441	B4	V488 Cru	+ .814	B3
AH Aur	— .306	F8	MN Cen	+ .543	B6	RZ Dra	— .259	A5
i Boo	— .572	G2	MP Cen	+ .476	A5	YY Eri	— .493	G5+G
SZ Cam	+ .431	B0	MR Cen	+ .592	A8	WW Gem	+ .093	B6
TU Cam	+ .467	A1	SU Cep	— .045	B8	AC Gem	+ .998	A0
RZ Cnc	+ 1.335	K2 + K5	VW Cep	— .556	G5	TT Her	+ .960	A2
RV CVn	— .569	F8	WY Cep	+ .093	A7	AK Her	— .376	F8
UW CMa	+ .642	O8 + O8	AH	+ .248	B0 + B0	U Her	+ .312	B3
YY CMi	+ .037	F5	TY Cet	— .199	F0	RY Ind	— .148	A5
X Car	+ .033	A0 + A1	RW Com	— .625	G4	RT Lac	+ .705	G9 +

Table 1 (continued).

Design	log *P*	Spect.	Design	log *P*	Spect.	Design	log *P*	Spect.
SW Lac	— .495	G3 + G3	U Peg	— .427	F3	U Sct	— .020	F0
VY Lac	+ .017	A2	SU Psc	+ .428	B5	RS Sct	— .178	F5
AW Lac	+ .057	A0	V Pup	+ .161	B3	V Ser	+ .538	B8
CN Lac	— .196	G3	TY Pup	— .237	A9	RS Ser	— .223	F5
V Lep	+ .029	A5	UZ Pup	— .100	A6 + A6	RZ Tau	— .381	F0 + F
RR Lep	— .091	A2	AU Pup	+ .053	A0	V Tri	— .233	A3
RV Lib	+ 1.029	G5	AV Pup	— .255	F8	W UMa	— .467	F8 + F
β Lyr	+ 1.111	B8	V525 Sgr	— .152	A2	AC Vel	+ .659	B8
V502 Oph	— .344	G0	V779 Sgr	— .352	F8	AY Vel	+ .210	B9
V566 Oph	— .288	F8	υ Sgr	+ 2.140	F2 + (B8)	BC Vel	+ .068	F8
VV Ori	+ .173	B2 + B8 ?	V453 Sco	+ 1.079	B0	AG Vir	— .192	A2
ER Ori	— .374	G1	V474 Sco	+ .209	B4	AH Vir	— .390	K0
FR Ori	— .054	A7	μ' Sco	+ .161	B3 + B6	BF Vir	— .194	A0
η Ori	+ .902	B1	RT Scl	— .292	A5			

In figure 1 the logarithm of the period is plotted against the spectral type of the teneous component. Only in a few cases it remains slightly doubtful which spectral type should be used. For RZ Cnc the spectral type of the teneous component was taken to be K5. With RX Cas, there can be no doubt that the density of the component of spectral type gG3 is less than that of spectral type A5. With *υ* Sgr the spectral type F2 has been adopted as that of the teneous component. Presumably this teneous component is a supergiant. In all other cases in which two spectra are given, the difference in spectral type is small. In the figure the mean of the two spectral types has been used.

Each separate star has been indicated by a filled circle. In the same way we have indicated the systems which are enumerated in table 2. This table contains the Algol variables for which the spectral types of both components are given and for which the difference between the spectral types of these components is ≦ 0.5 spectral class.

Table 2. *Algol Variables of which the Spectral Types of both Components are known and for which the Difference Between the Spectral Types is ≦ 0.5 Spectral Class.*

Design	log *P*	Spect.	Design	log *P*	Spect.	Design	log *P*	Spect.
WW Aur	+ 0.401	A7 + A7	CC Cas	+ .528	O8 + O8	WZ Oph	+ .621	G0 + G
AR Aur	+ .616	A0 + A0	WX Cep	+ .529	A2 + A5	AG Per	+ .308	B3 + B
EO Aur	+ .609	B3 + B3	U CR B	+ .538	B5 + A0	V356 Sgt	+ .949	B9 + A
β Aur	+ .598	A2 + A2	Y Cyg	+ .477	O9 + O9	V472 Sco	+ 2.320	K5 + M
SV Cam	— .227	G3 + G3	YY Gem	— .089	K6 + K6	W Scu	+ 1.013	B0 + B
SW Cma	+ 1.004	A8 + A8	AR Lac	+ .297	K0 + G5	SV Tau	+ .336	B9 + A
RW Cap	+ .530	A3 + A4	UV Leo	— .222	G0 + G2	Z Vul	+ .389	B3 + B
TV Cas	+ .258	A0 + A0	AO Mon	+ .274	B3 + B5			
TW Cas	+ .456	B9 + A0	U Oph	+ .225	B5 + B5			

When plotting this group of stars, the spectral type adopted for AR Lac was K0 and that for V 472 Sco was M0. In the figure a third group of stars has been indicated, viz. the spectroscopic double stars

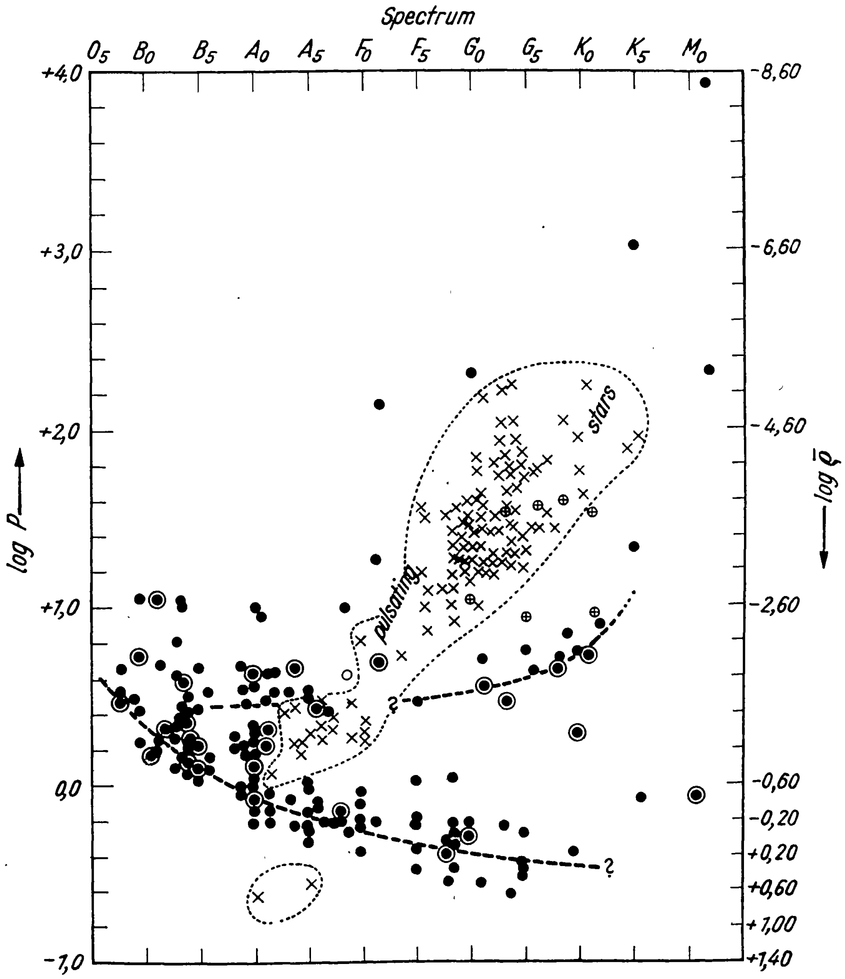

Fig. 1.

for which all elements are given and from which the equation 1 has been derived. These stars have been enumerated in a previous paper [Z. Astrophysik **31**, 256 (1952), table I]. This table is not repeated here. In the figure they are indicated by a dot surrounded by a circle.

Finally in the figure I have also plotted the Algol variables which are enumerated in table 3. Of these systems the spectral types of both components have been observed and appear to be different. With the stars enumerated in table 3 we are reasonably certain which spectral type corresponds to the teneous component. In the table this spectral type has been underlined. However, for our present purpose the points obtained from these stars must be considered as second class determinations only. In the figure they are indicated by open circles with a cross.

Table 3. *Algol Variables for which the Spectral Types of both Components are Observed and for which the Spectral Difference is Large.*

Design	log P	Spect.	Design	log P	Spect.
RT And	—0.202	$\underline{\text{G0}}$ + K1	VV Cep	+ 3.871	$\underline{\text{M2}}$ + B
TW And	+ .615	dF0 + $\underline{\text{G6}}$	ZZ Cep	+ .330	$\underline{\text{B7}}$ + dF0
RS Ari	+ .944	$\underline{\text{G5}}$ + F9	UZ Cyg	+ 1.496	A3 + $\underline{\text{K1}}$
ζ Aur	+ 2.988	$\underline{\text{K5}}$ + B9	WW Dra	+ .666	G2 + $\underline{\text{K0}}$
SS Cam	+ .683	$\underline{\text{G1}}$ + F5	RZ Eri	+ 1.594	$\underline{\text{G8}}$ + F5
RS CVn	+ .681	F4 + $\underline{\text{G8}}$	AW Her	+ 945	$\underline{\text{K2}}$ + G4
SX Cas	+ 1.563	A6 + $\underline{\text{G6}}$	RW Per	+ 1.121	A5 + g$\underline{\text{G0}}$
AQ Cas	+ 1.068	$\underline{\text{B3}}$ + B9	RW UMa	+ .865	F9 + $\underline{\text{G9}}$
U Cep	+ .396	$\underline{\text{B8}}$ + G2			

There certainly is an additional number of two spectra stars, but with these additional pairs there is no indication whatever, which spectrum is that of the teneous component. Therefore these stars have not been included in table 3. Even with the pairs in table 3 in several cases the choice is dubious.

§ 3. Pulsating Stars.

Another group of variables for which the periods are directly related to the densities, are the Cepheids and cluster type variables. For these stars the relation is approximately expressed by the equation (2)

$$\log P = -{}^1\!/_2 \log \bar{\varrho} - 0.939\,. \tag{2}$$

By comparing (1) and (2) we see that the mean density of a Cepheid being equal to that of the teneous component of a binary system, the logarithm of the period is 0.638 smaller.

Consequently when the Cepheids are plotted in figure 1, for having the same scale of densities the logarithm of the periods of the Cepheids must be increased by + 0.638. Then for both binaries and the Cepheids the scale of densities is identical and as indicated on the right side of the figure.

Table 4. *Cepheids used in the Present Investigation.*

Design	Spect.	$\overline{\text{Spect.}}$	log P + .64	Design	Spect.	$\overline{\text{Spect.}}$	log P + .64
SW And	A3—F8	F0.5	+ .29	RT Aur	F1—G5	F8	+ 1.21
XX And	A3—F5	A9	+ .50	RX Aur	G0—K0	G5	+ 1.70
CY Aqr	B8—A3	A0.5	— .58	SY Aur	G0—G2	G1	+ 1.64
U Aql	G0—G6	G3	+ 1.49	RS Boo	B8—F0	A4	+ .22
SZ Aql	G0—K5	G7.5	+ 1.87	RU Boo	K0—M2	K6	+ 1.99
TT Aql	F8—K0	G4	+ 1.78	RX Boo	G2—K2	G7	+ 1.54
η Aql	F6—G4	G0	+ 1.50	W CVn	A6—F6	F1	+ .38
S Ara	A3—F5	A9	+ .30	U Car	F9—K3	G6	+ 2.23
X Ari	A0—A7	A3.5	+ .45	V Car	G0—K2	G6	+ 1.47
Y Aur	F8—G5	G1.5	+ 1.23	SX Car	F5—G8	G1.5	+ 1.33

Table 4 (countinued).

Design	Spect.	$\overline{\text{Spect.}}$	log P + .64	Design	Spect.	$\overline{\text{Spect.}}$	log P + .64
UX Car	F4—G5	F9.5	+ 1.21	RR Leo	A0—F4	A7	+ .31
VY Car	F9—K0	G4.5	+ 1.94	RR Lyr	A2—F0	A6	+ .39
l Car	F8—K0	G4	+ 2.19	T Mon	F7—K1	G4	+ 2.07
SU Car	F5—F7	F6	+ .92	SV Mon	F8—K5	G6.5	+ 1.81
SW Car	G0—K0	G5	+ 1.38	R Mus	F9—G6	G2.5	+ 1.51
SY Car	F8—G5	G1.5	+ 1.25	S Mus	F8—G4	G1	+ 1.62
TU	F5—G2	F8.5	+ .97	S Nor	F8—G2	G0	+ 1.63
V Cen	F5—G5	G0	+ 1.38	U Nor	G0—K5	G7.5	+ 1.74
UZ Cen	F2—G5	F8.5	+ 1.16	Y Oph	F9—G5	G2	+ 1.87
XX Cen	F6—K0	G3	+ 1.68	BF Oph	F8—K2	G5	+ 1.25
AY Cen	F8—K0	G4	+ 1.36	RS Ori	F2—G0	F6	+ 1.52
AZ Cen	F3—G0	F6.5	+ 1.15	X Pav	F5—G5	G0	+ 1.60
V 378	F5—G5	G0	+ 1.45	DY Pav	A3—A9	A6	— .50
V 381	F6—G7	G1.5	+ 1.35	SV Per	F8—K0	G4	+ 1.69
RZ Cep	A0—A3	A1.5	+ .13	X Pup	F5—K2	G3.5	+ 2.06
δ Cep	F5—G2	F8.5	+ 1.37	RS Pup	G5—K7	K1	+ 2.26
RR Cep	A5—F0	A7.5	+ .38	S Sge	F6—G5	G0.5	+ 1.56
R Cru	F6—G7	G1.5	+ 1.40	U Sgr	F7—G5	G1	+ 1.47
S Cru	F5—G5	G0	+ 1.31	W Sgr	F2—G6	F9	+ 1.52
T Cru	G1—G5	G3	+ 1.47	X Sgr	F5—G9	G2	+ 1.49
X Cru	G0—K0	G5	+ 1.43	Y Sgr	F6—G5	G0.5	+ 1.40
X Cyg	F7—G8	G2.5	+ 1.85	WZ Sgr	G3—K6	G9.5	+ 1.98
SU Cyg	F0—G1	F5.5	+ 1.22	XX Sgr	F8—G8	G5	+ 1.45
SZ Cyg	F8—G8	G3	+ 1.82	YZ Sgr	G0—G7	G3.5	+ 1.62
TX Cyg	F5—G6	G0.5	+ 1.81	AP Sgr	F6—G5	G0.5	+ 1.34
UY Cyg	A5—F0	A7.5	+ .39	BB Sgr	F8—G5	G1.5	+ 1.46
VY Cyg	F6—G1	F8.5	+ 1.54	V 350 Sgr	F5—G4	F9.5	+ 1.35
VZ Cyg	F5—G5	G0	+ 1.33	RV Sco	F5—G5	G0	+ 1.42
XZ Cyg	A0—A8	A4	+ .31	RY Sco	F9—G7	G3	+ 1.95
BZ Cyg	F8—G5	G1.5	+ 1.64	X Scu	F5—K2	G3.5	+ 1.26
CD Cyg	F8—K0	G4	+ 1.87	Y Scu	G5—K7	K1	+ 1.65
DT Cyg	F5—F7	F6	+ 1.04	Z Scu	G0—M0	K0	+ 1.75
MW Cyg	F8—G1	F9.5	+ 1.41	RU Scu	G5—M5	K5	+ 1.94
V 386	F5—G1	F8	+ 1.36	SS Scu	F9—G5	G2	+ 1.20
β Dor	F2—F9	F5.5	+ 1.63	ST Tau	F5—G5	G0	+ 1.25
SU Dra	A2—A5	A3.5	+ .46	SW Tau	A7—F2	A9.5	+ .84
RX Eri	A3—F0	A6.5	+ .51	SZ Tau	F6—F9	F7.5	+ 1.14
W Gem	F6—G5	G0.5	+ 1.54	R Tr A	F6—G4	G0	+ 1.17
RZ Gem	F5—G5	G0	+ 1.38	S Tr	F6—G8	G2	+ 1.44
ζ Gem	F7—G3	G0	+ 1.63	U Tr	F4—G2	F8	+ 1.05
VX Her	A3—F0	A6.5	+ .30	TV Vel	G0—G5	G2.5	+ 1.31
BL Her	A8—G0	F4	+ .72	V Vel	F8—G5	G1.5	+ 1.28
V Ind	A0—F2	A6	+ .32	RY Vel	G2—K2	G7	+ 2.09
V Lac	F2—G5	F8.5	+ 1.34	SV Vel	F8—K0	G4	+ 1.75
X Lac	G1—G5	G3	+ 1.38	T Vul	F5—G1	F8	+ 1.29
Z Lac	F6—G6	G1	+ 1.68	U Vul	F8—K0	G4	+ 1.54
RR Lac	F5—K0	G2.5	+ 1.45	X Vul	G0—K5	G7.5	+ 1.44
BG Lac	F7—G4	G0.5	+ 1.37	SV Vul	F7—K0	G3.5	+ 2.29

In the figure only those Cepheids are plotted for which in the New Catalogue of variable stars (l. c.) the total range of spectral fluctuation is given and which are indicated as Cep or C1.

Just as with the previous tables in table 4 only those values are inserted which are used in the present investigation.

In the figure the mean values of the spectral type as given in table 4 are plotted against the corresponding values ($\log P + 0.64$). The resulting points are indicated by crosses.

§ 4. Discussion of the Diagram.

Obviously figure 1 is a distorted form of the Russell-Hertzsprung diagram in which instead of spectral type and absolute magnitude as coordinates the spectral type and the (mean) density have been used. The main sequence is clearly indicated all the way from K0 to O5. The giant series is clearly indicated between K0 and G0 but is very indistinct in the Hertzsprung gap between about F8 and A6. The early A stars still seem to fall apart into two distinct groups corresponding to the main sequence and the giant series respectively. Only with the A0 stars the two groups seem completely to have immersed into one another.

In the figure the separation between giants and dwarfs among the binary stars may be more pronounced than is actually the case. This separation may have been accentuated by various selection effects. The systems along the main sequence are short period variables which may be more easily detected than those of longer period. The systems along the giant series usually are stars of considerable apparent brightness, which as a group have been more intensively observed than stars of fainter brightness. In the diagram the apparent ridge lines of the main sequence and of the giant sequence are indicated by dotted curves. The mean densities corresponding to the various spectral types as read from these curves are indicated in table 5. There is however a considerable spread of the individual values of the density around this mean.

Table 5. *Mean Values of Density* ($\odot = 1$).

Spects.	main sequence	giant series
O5	0.015 ?	—
B0	0.05	—
B5	0.14	—
A0	0.33	0.14
A5	0.55	— ?
F0	0.87	— ?
F5	1.26	0.035
G0	1.58	0.025
G5	1.74	0.017
K0	2.09	0.009
K5	—	0.0019 ?

From the distribution of the points representing the Cepheids it appears that the Cepheids have densities smaller than those of the giant stars of corresponding spectral type. In the Cepheid belt almost no double stars occur. The few stars which occur in this belt all are uncertain determinations. The Cepheid belt intersects the giant series near the Hertzsprung gap and in this gap almost no Cepheids occur. Obviously with this arrangement the well known intervall between the short and long period Cepheids coincides with the Hertzsprung gap. The Cepheid belt continues below the giant series. Here the Cepheids almost exclusively fall into the open region between the bright and the faint stars of spectral type A.

At the place where the Cepheid belt intersects the main sequence, no pulsating stars occur, but a few may still occur below this sequence. Therefore here also we find that no or almost no binary systems occur in the region of stellar instability, while conversely no Cepheids occur outside this region.

A small group of binary systems have densities smaller than those of the Cepheids. Tentatively this group might be identified with the supergiants.

Monrovia, October 1952.

Literature.

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